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SEVERELY ECCENTRIC LOADS ON ROUND FOOTINGS.(U)  
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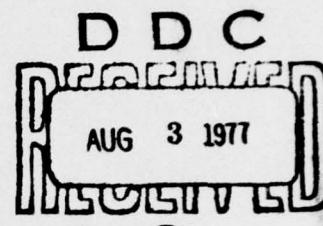
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## SEVERELY ECCENTRIC LOADS ON ROUND FOOTINGS

DIRECTORATE OF ENGINEERING  
AIR FORCE CIVIL ENGINEERING CENTER  
TYNDALL AFB, FL. 32403

JANUARY 1977



FINAL REPORT FOR PERIOD SEPTEMBER 1976 TO  
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AIR FORCE CIVIL ENGINEERING CENTER  
(AIR FORCE SYSTEMS COMMAND)

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PREFACE

This report was prepared by the Air Force Civil Engineering Center (AFCEC), Tyndall Air Force Base, Florida, under job order 20545007. This report summarizes work done between September and December 1976. Captain Dennis Morrison was the project engineer.

This report has been reviewed by the Information Office (OI) and is releasable to the National Technical Information Service (NTIS) where it will be available to the general public including foreign nations.

This technical report has been reviewed and is approved for publication.

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SECTION I  
INTRODUCTION

Round footings can be used to support eccentric loads where the location of the eccentricity may vary circumferentially around the footing. An example of such a loading condition would be the ground line forces induced by wind on a wind turbine mounted on unguyed precast pole (Figure 1). The direction of the wind may vary, thus the location of the eccentricity may vary.

Conventional analysis of rectangular footings to determine bearing pressure under eccentric loads applies to round footings with considerations for the change in geometric shape. Consider the following example.

Assume a round footing with an eccentric load as in Figure 2(a). If the entire footing is going to bear on the soil, the compressive stress ( $P/\pi R^2$ ) at the extreme edge of the footing away from the load (Figure 2(b)) must be greater than or equal to the bending (tipping) stress ( $4Pe/\pi R^3$ ) (Figure 2(c)). If equality exists, the pressure varies from  $(P/\pi R^2) + (4Pe/\pi R^3)$  on one edge to zero on the other (Figure 2(d)). Using the equality as a boundary and solving for the eccentricity yields

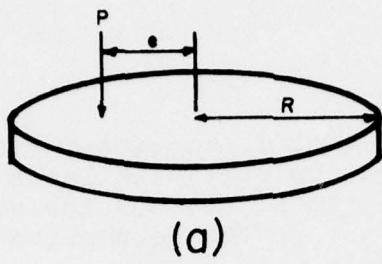
$$P/\pi R = 4Pe/\pi R^3 \quad \text{or} \quad e = R/4$$

which reveals that the load must lie within the middle one-fourth of the diameter for the condition of bearing across the entire footing. It should be assured, of course, that the maximum stress in the soil never exceeds the allowable soil bearing pressure.

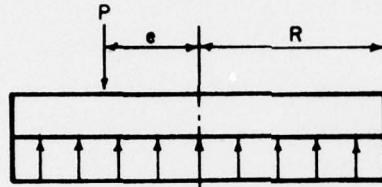
This report exhibits the considerations for sizing a round footing when the eccentricity is outside of the boundary shown by the equation. That is, when the load on a round footing is outside the middle one-fourth of the diameter. Design of such footings would probably not be a common approach but conditions such as availability of round form work, property boundary line problems, or checking an existing design may warrant use of this report. Another use may evolve for checking of bearing pressures for unusual wind



Figure 1. Wind Turbine on Unguyed Precast Pole

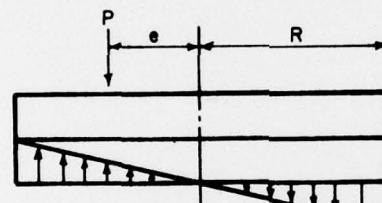


(a)



$$\sigma_c = P / \pi R^2$$

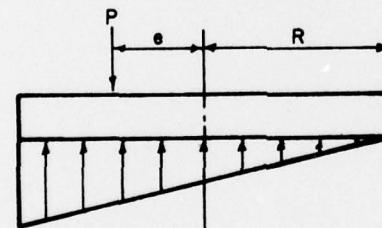
(b)



$$\sigma_b = -4Pe/\pi R^3$$

(c)

$$\sigma_T = P/\pi R^2 + 4Pe/\pi R^3$$



$$\sigma_T = 0$$

(d)

Figure 2. Eccentric Load on Round Footing

loads such as the previously mentioned wind turbine. Conventional analysis neglects wind loads for examination of bearing pressures for footings, but the very nature of a wind turbine suggests that the assumption of neglecting windloads may not apply.

## SECTION II

### DEVELOPMENT OF EQUATIONS

For a round footing with a severely eccentric load (eccentricity outside the middle one-fourth) the geometric analysis exhibited in the previous section becomes more complex, however the basic assumptions of footing rigidity and linear soil pressure distribution will still be used. The solution of the equations below and their graphical interpretations serve as a tool to avoid undesirable iterative solutions for each design case. Checks for shear, bending and other reinforcement must be performed after selection of footing size, but that analysis is not included in this report.

Development of the equations begins, as shown in Figure 3, with a round footing of generalized radius  $R$  with a load  $P$  applied at a distance  $e$  from the centroid of the footing. The eccentricity ( $e$ ) is defined as being greater than or equal to  $R/4$  and the maximum pressure is  $q$ . The distance to the line of zero pressure is measured from the centroid of the footing orthogonal to the line and is defined as  $b$ . Basic static equilibrium must apply, hence the resultant of the soil pressure must be (1) equal in magnitude, (2) opposite in sign, and (3) colinear with the applied load. For conditions (1) and (2)

$$P = - \int Q(x) dx \quad (1)$$

where  $Q(x)$  is the pressure exerted on the footing by an infinitesimal strip of soil  $dx$  wide a distance  $x$  from the center of the footing. Figure 4 shows the strip of soil and yields

$$Q(x) = [2q/(R+b)](x-b) \sqrt{R^2-x^2} dx$$

therefore

$$P = [2q/(R+b)] \int_{-R}^b (x-b) \sqrt{R^2-x^2} dx$$

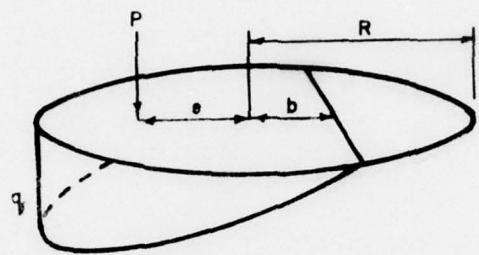


Figure 3. Generalized Round Footing

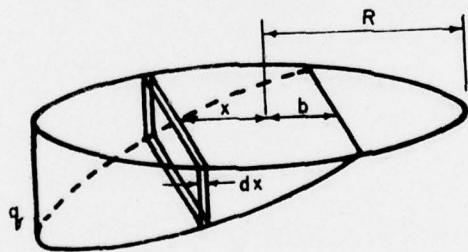


Figure 4. Pressure on Infinitesimal Strip of Soil

Relatively simple integration and substitution of limits give

$$P = [-q/(R+b)] \cdot 2/3(R^2 - b^2)^{3/2} + R \cdot b \left[ \frac{(R^2 + b^2)^{1/2}}{R^2} + \sin^{-1} b/R + 1.57 \right] \quad (2)$$

To maintain the resultant of the soil pressure collinear with the eccentric load (condition 3), the distance to the centroid of the volume defined by the right hand side of equation (1) must be  $e$ , so that

$$e = \frac{\int x Q(x) dx}{\int Q(x) dx} \quad (3)$$

The denominator of equation (3) is equation (2) and the numerator becomes

$$\left[ 2q/(R+b) \right] \int_{-R}^b (x^2 - bx) \cdot R^2 - x^2 dx$$

again integration and substitution of limits gives

$$x \cdot Q(x) dx = \left[ 2q/(R+b) \right] \left\{ b/12(R^2 - b^2)^{3/2} + R^4/8 \left[ b/R^2(R^2 + b^2)^{1/2} + \sin^{-1}(b/R) + 1.57 \right] \right\} \quad (4)$$

Division of equation (4) by equation (2) gives the solution to equation (3) as

$$e = \frac{bd^3/12 + R^4/8(bd/R^2 + \sin^{-1} b/R + 1.57)}{d^3/3 + R^3b/2(d/R^2 + \sin^{-1} b/R + 1.57)} \quad (5)$$

where

$$d = \sqrt{R^2 - b^2}$$

The simultaneous solution of equations (2) and (5) for  $R$  or  $e$  is not easily obtained and the next section provides an approach to make these equations useful.

### SECTION III

#### RESULTS

Equations developed in the previous section are not readily useable in that form, but with the development of computers and programmable calculators, R and b can be easily varied over a range of interest and e computed. Note that e is not a function of q and that P varies linearly with q so that a normalized P ( $P/q$ ) can be found using q equal to unity while varying R and b. Figure 5 is a cartesian plot of P normalized versus e. Note the limit of  $e = R/1.7$ . That is where the zero pressure line crosses the centroid of the footing ( $b = 0$ ). Beyond that arbitrary point, it is felt, that the basic assumptions of footing rigidity and linear soil pressure distribution are not as reliable.

Design Aid Steps for use of Figure 5 follows:

- (1) Choose radius (R) in feet and depth of footing.
- (2) Compute  $e = M/P$ , where M is applied moment in foot-pounds and p is axial load at the bottom of the footing in pounds. Include weight of footing if applicable.
- (3) With R and e enter Figure 5 and find P normalized ( $P/q$ ).
- (4) Multiply  $P/q$  by the allowable soil bearing pressure in pounds per square foot to obtain  $P_a$ .
- (5) If  $P_a > P$ , then R is adequate. The next smaller R may be tried. If  $P_a < P$ , then R is inadequate and a larger R must be chosen.

The above is only a suggested use of Figure 5 and the user may use the chart according to need. The only unit required is the use of feet throughout. Feet may be converted to meters by multiplying by 0.305 and square feet to square meters by multiplying by 0.0929.

This report has shown that aids for design of round footings with eccentric loads can be developed. The basic principal can be extended to other shapes such as an eccentric load on the diagonal of a rectangular footing. Proper

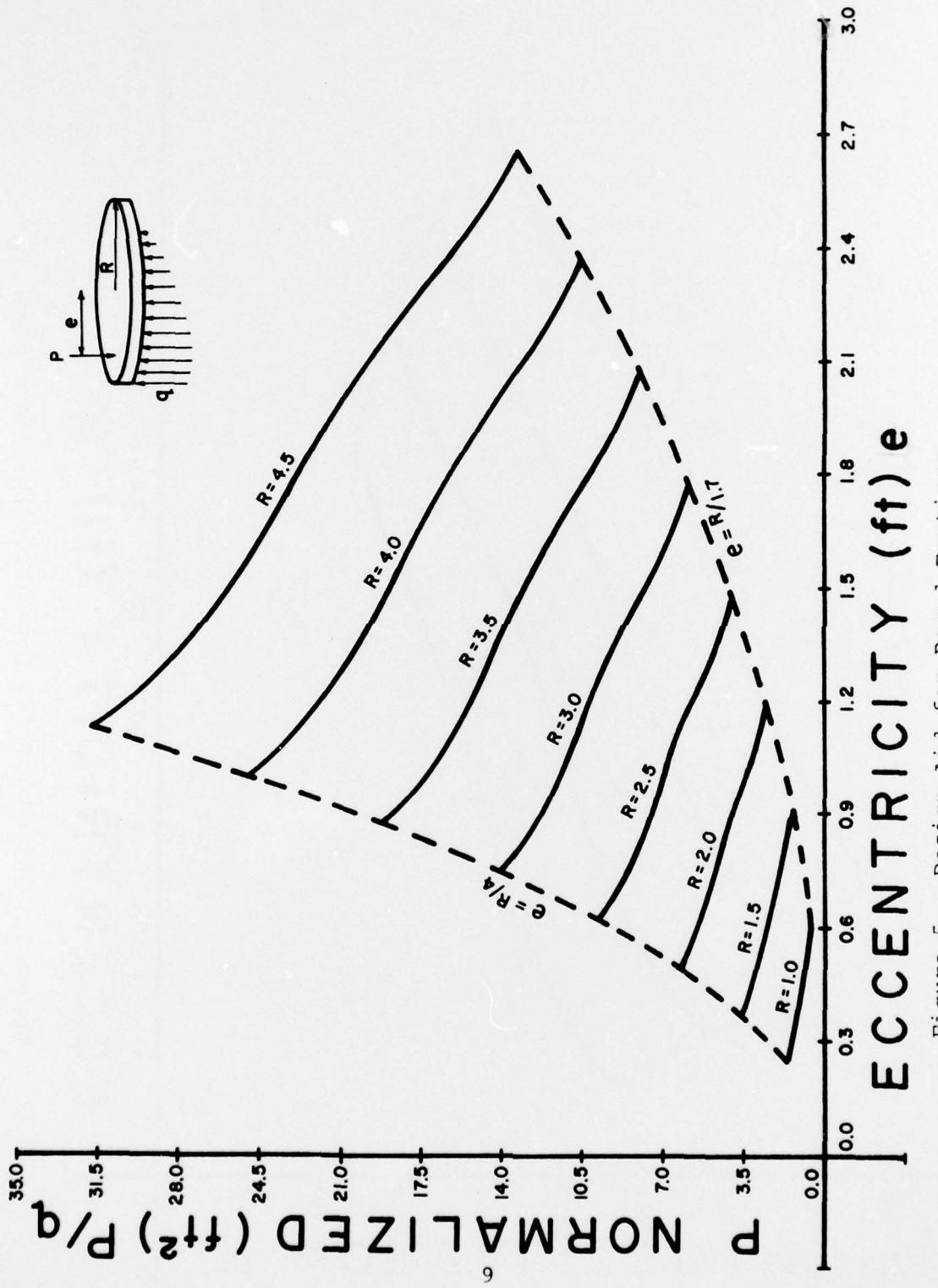


Figure 5. Design Aid for Round Footings

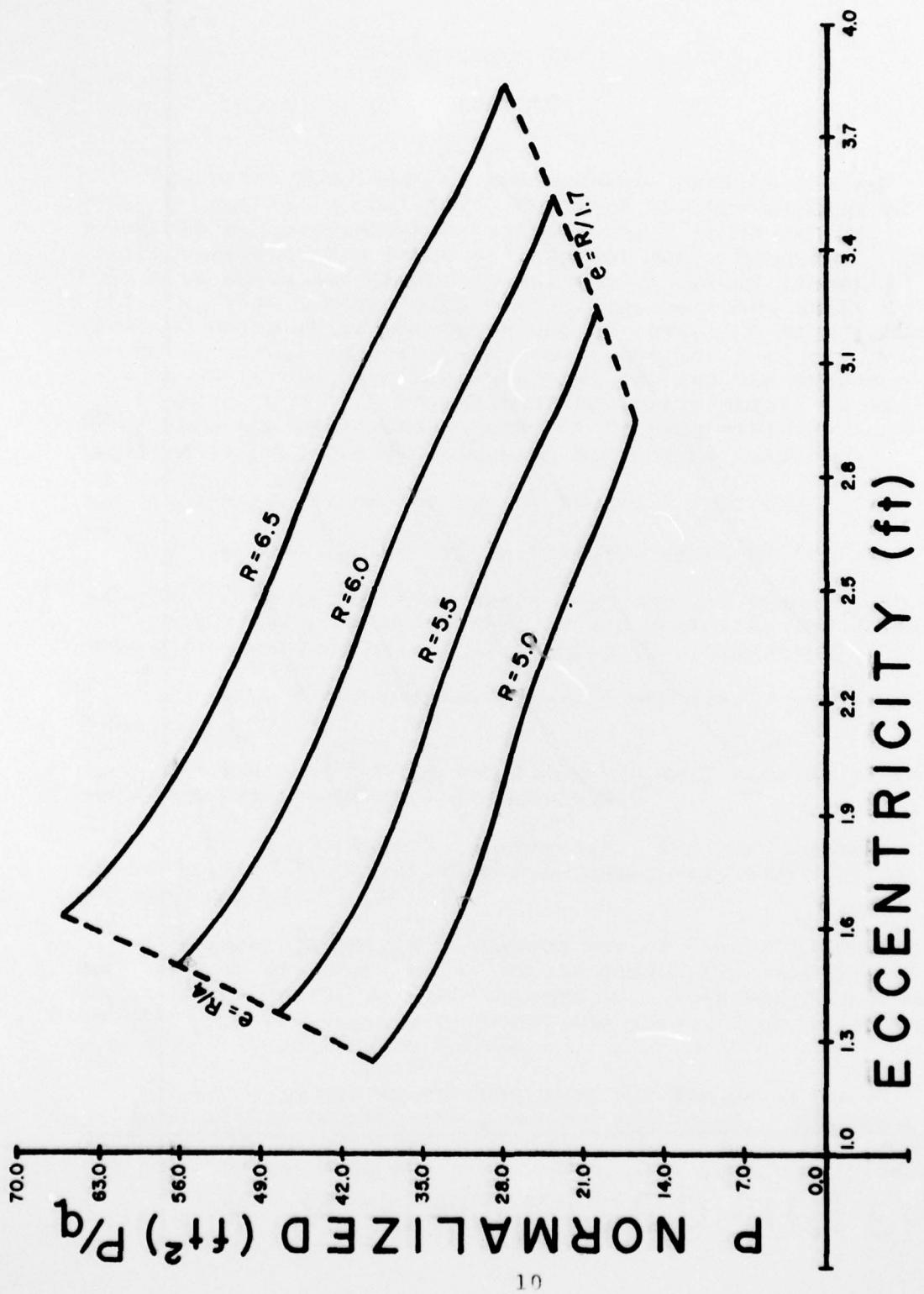


Figure 5. Design Aid for Round Footings (Continued)

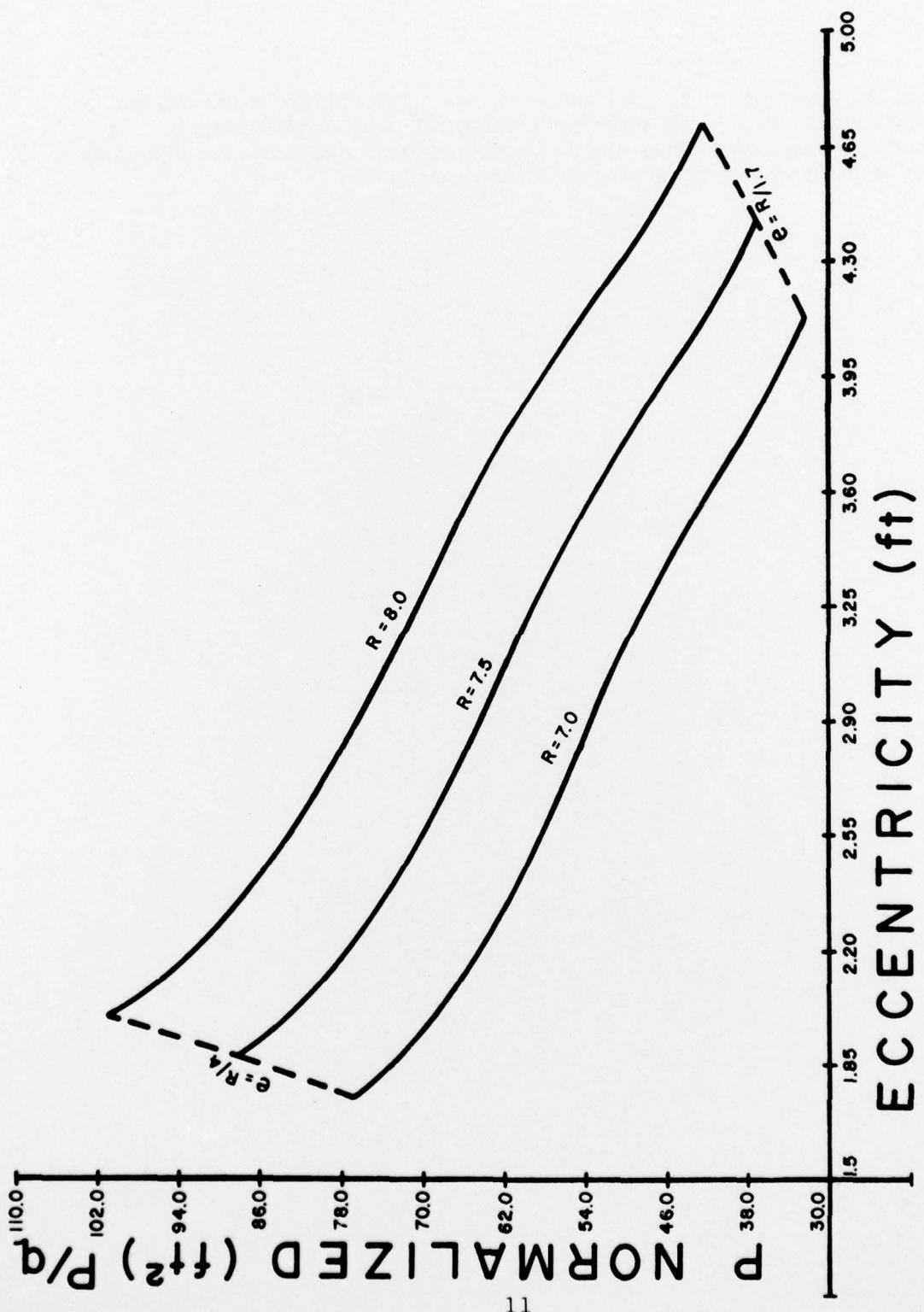


Figure 5. Design Aid for Round Footings (Concluded)

application of this design aid, as with other such aids, requires a thorough understanding of its development. To curtail improper use, personnel lacking derivative knowledge are discouraged from using these aids.

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